Problem Description of the 11th China Trajectory Optimization Competition

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Released: 20 September, 2020

1 Problem Description

In recent years, most aerospace countries in the world have begun to rapidly develop the responsive space technology. The responsive space technology is intended to make rapid response for local emergencies, and it can be considered from two aspects: 1) using inventory technology to achieve satellite rapid assembly, rapid testing, rapid launch, etc.; 2) using orbital maneuver to adjust the ground track of spacecraft to realize fast earth observation mission for specified ground site, which does not need to launch a new spacecraft. When earthquakes, forest fires or other natural disasters occur, it needs more financial cost to launch new satellites, and it takes a certain time for developing and launching new satellites. It is a more economical and fast way to adjust the ground track by using orbit maneuvers, so that the earth observation mission for the specified ground site can be completed in a short time. Thus, it is of great significance to realize the earth observation and data transmission tasks for multiple ground sites and moving targets, respectively. The topic of the 11th China Trajectory Optimization Competition (CTOC 11) is: **orbit design and maneuver optimization of two satellites for complex observation tasks with a large number of targets on the Earth**.

In this competition, there are two earth observation satellites. One is equipped with an infrared camera and the other with an optical camera. They are called the infrared satellite and the optical satellite, respectively. By applying reasonable orbit maneuvers to the two satellites, the participants need to complete the observation task of 200 stationary targets and 5 moving targets designated on the Earth (see Fig. 1); moreover, each observation satellite must conduct a data transmission with one of the relay satellites after observing 30 targets at most. According to different target points and optimization indexes, this competition is divided into the following three parts:

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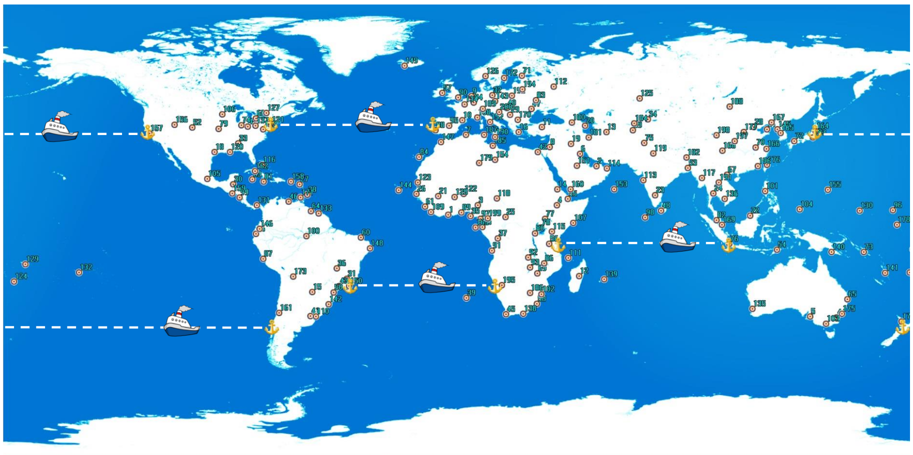

Figure 1: Target distribution diagram.

(1) Fast Observation of Stationary Targets

In order to rapidly obtain the data information of ground stationary targets, two observation satellites are required to cooperatively complete the observation of 200 stationary targets in the shortest time within 60 days at the beginning of the mission. The longitudes and latitudes of the ground targets are shown in Attachment 1.

(2) Accurate Revisit Observation of Stationary Targets

In order to verify the revisit ability of observation satellites, two observation satellites are required to complete two observations with exact revisit interval for the 200 stationary ground targets within 120 days. The revisit interval is set to be 30 days, that is, the time interval between two observations for each target point is required to be 30 days as far as possible.

(3) Multiple Uniform Observation of Moving Targets

There are five moving targets on the sea, all of which move back and forth at a constant speed on the route shown in Fig. 1. In order to verify the observation ability for the moving targets, two observation satellites are required to observe five moving targets many times in 60 days. During the mission, the observation times of all moving targets should be as many and uniform as possible. See Attachment 1 for the route data.

2 Optimization Index and Evaluation Standard

2.1 Single Observation Score

The camera of the observation satellite always points to the center of the earth, and the line-of-sight angle between the observation satellite to the earth center and the observation satellite to the target point is defined as

$$\gamma = \arccos\left(\frac{\boldsymbol{r}_{obs} \cdot (\boldsymbol{r}_{obs} - \boldsymbol{r}_{tar})}{\|\boldsymbol{r}_{obs}\| \cdot \|\boldsymbol{r}_{obs} - \boldsymbol{r}_{tar}\|}\right), \quad \boldsymbol{r}_{obs} \cdot \boldsymbol{r}_{tar} > 0 \tag{1}$$

where $\boldsymbol{r}_{tar}$ and $\boldsymbol{r}_{obs}$ are the position vectors of the ground site and the observation satellite in the Earth Centered Inertial (ECI) frame, respectively, and $\boldsymbol{r}_{tar}$ can be obtained in Appendix B3. When the target point enters the field of view of the camera ($\gamma \leq 2.5°$), it is considered that one observation is completed (see Fig. 2).

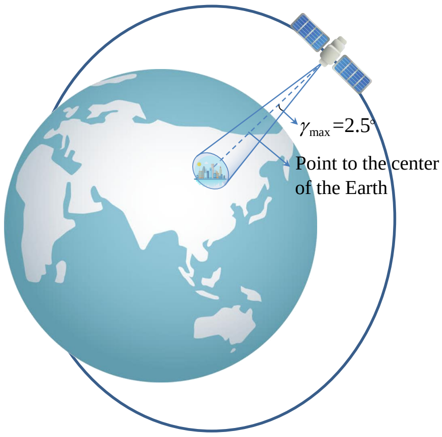

Figure 2: Geometric comprehension for earth observation.

The single observation score of any target point is

$$s = C(5 - 0.01h) \tag{2}$$

where h is the orbital height of the observation satellite during observation, and $h \in [200, 500]$ km; C is the camera coefficient. For different cameras in the competition, $C = 1$ is set for the optical satellite, and $C = 0.8$ is set for the infrared satellite.

2.2 Evaluation Criteria

(1) Task 1

Task 1 requires the observation satellite to complete the observation task for all ground stationary targets in the shortest time. The earlier the target is observed, the higher the score will be. The evaluation index is

$$W_1 = A_1 \frac{T_1}{t_{1end}} \sum_{i=1}^{200} \frac{s_i}{t_{1end}/86400 + (t_i/86400)^2} \tag{3}$$

where W_1 is the score of the Task 1, $A_1 = 14$ is the adjustment coefficient of the Task 1, s_i is the observation score of the target point i, t_i(s) is the observation time for the target point i, t_{1end}(s) is the time when the last target point is observed, and $T_1 = 60 \times 86400$ s is the deadline of the Task 1. If the observation of all target points is not completed within 60 days, $W_1 = 0$.

(2) Task 2

Task 2 requires the observation satellite to accurately revisit the ground target point according to the desired time interval. The evaluation index is

$$W_2 = A_2 \sum_{i=1}^{200} P_i \bar{s}_i \tag{4}$$

where W_2 is the score of the Task 2, $A_2 = 4$ is the adjustment coefficient of the Task 2, $\bar{s}_i = (s_{i1} + s_{i2})/2$ is the average score of two observations for the target point i, and $P_i(\Delta t_i)$ is the revisit coefficient of target point i,

$$P_i(\Delta t_i) = \frac{1}{3\sqrt{2\pi}} \exp\left[-\frac{(\Delta t_i/86400 - 30)^2}{18}\right] \tag{5}$$

where Δt_i(s) is the time interval between two observations for the target point i. If the revisit of all target points is not completed within 120 days, $W_2 = 0$.

(3) Task 3

In the multiple uniform observation task of moving target points, the number of observations and uniformity are the standard, and the evaluation index is

$$W_3 = \frac{A_3}{(1 + 0.1\sigma_3)\sqrt{N_{tot}}} \sum_{i=201}^{205} \sum_{j=1}^{N_i} s_{ij} \tag{6}$$

where W_3 is the score of the Task 3, $A_3 = 10$ is the adjustment coefficient of the Task 3, s_{ij} is the j-th observation score of the moving target point i, N_i is the number of observations of the moving target point i, $N_{tot} = \sum_{i=201}^{205} N_i$ is the total number of observations of all moving target points in Task 3, and σ_3 is the standard deviation of the observation number of each moving target point,

$$\sigma_3 = \left[\frac{1}{5} \sum_{i=201}^{205} (N_i - N_{tot}/5)^2\right]^{\frac{1}{2}} \tag{7}$$

Finally, the whole task score is

$$W_{tot} = B(W_1 + W_2 + W_3) \tag{8}$$

where B is the time coefficient,

$$B = 1 + 0.3\left(\frac{t_{end} - t_{submission}}{t_{end} - t_{start}}\right) \tag{9}$$

where $t_{submission}$ is the result submission time, t_{start} is the opening time of the submission, and t_{end} is the competition end time.

3 Satellite System Parameters and Orbit Dynamics Model

3.1 Satellite System Parameters

There are two observation satellites and four relay satellites in the task. The two observation satellites are used to carry out the imaging task of ground target points, and they are equipped with the infrared camera and the optical camera, respectively. The dry weight of each observation satellite is 300 kg, and the initial fuel load is 250 kg for each observation satellite. The initial orbits of the four relay satellites are given [see Eq. (14)], and there are no orbit maneuvers during the task. The relay satellites are only used to receive the imaging data of the observation satellites. Moreover, the orbital planes of the four relay satellites are evenly distributed in the space, and they are in large elliptical orbits with perigee height about 400 km.

3.2 Orbital Dynamic Model of Observation Satellites

The gravity, J_2 perturbation and atmospheric drag are considered in the orbital dynamics for the observation satellite, and the effects of other perturbations are ignored. In the J2000 ECI frame, the orbital dynamic equation of the observation satellite is

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\boldsymbol{r} + \boldsymbol{a}_{J_2} + \boldsymbol{a}_d \tag{10}$$

where $\boldsymbol{r} = [x, y, z]^{\mathrm{T}}$ is the position vector of the observation satellite in the ECI frame, $r = \|\boldsymbol{r}\|$ is the radius, μ is the gravitational parameter of the Earth, $\boldsymbol{a}_{J_2}$ is the J_2 perturbation acceleration caused by the earth's oblateness, and $\boldsymbol{a}_d$ is the atmospheric drag acceleration (see Appendix A).

The high-thrust chemical thrusters are used for the two observation satellites, and the maneuver process is considered as an instantaneous impulse. The impulse vector in the ECI frame is $\Delta\boldsymbol{v}$, and the times before and after the impulse are denoted by t^- and t^+, respectively. The position and velocity vectors satisfy the following equation:

$$\begin{cases} \boldsymbol{r}(t^+) = \boldsymbol{r}(t^-) \\ \boldsymbol{v}(t^+) = \boldsymbol{v}(t^-) + \Delta\boldsymbol{v} \end{cases} \tag{11}$$

The mass change of the observation satellite before and after the impulse satisfies

$$m(t^+) = m(t^-)\exp\left(-\frac{\Delta v}{g_0 I_{sp}}\right) \tag{12}$$

where $\Delta v = \|\Delta\boldsymbol{v}\|$ is the impulse magnitude, $I_{sp} = 300$ s is the specific impulse, and g_0 is the gravity acceleration at the earth's sea level (see Table 2).

3.3 Orbital Dynamic Model of Relay Satellites

Since the relay satellites are in large elliptical orbits far from the Earth surface, the effect of atmospheric drag is weak and it is ignored. In the process of its motion, only the effects of gravity and J_2 perturbation are considered, and the effects of other perturbations are ignored. In the J2000 ECI frame, the orbital dynamic equation of the relay satellite is

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\boldsymbol{r} + \boldsymbol{a}_{J_2} \tag{13}$$

4 Constraint Conditions

(1) Initial State Constraint

At the beginning of the task, the two observation satellites are in a circular orbit with the orbital altitude of 350 km ($a = R_E + 350\text{km}$, $e = 0$). Other orbit elements are to be selected, but the initial orbital plane (orbit inclination i and ascending node right ascension Ω) of the two observation satellites should be the same.

At the beginning of the task, the instantaneous orbital elements of the four relay satellites are

$$[a, e, i, \Omega, \omega, f] = \begin{cases} [16763\ \text{km}, 0.5957, 63.4°, 0°, 270°, 0°] \\ [16763\ \text{km}, 0.5957, 63.4°, 90°, 270°, 0°] \\ [16763\ \text{km}, 0.5957, 63.4°, 180°, 270°, 0°] \\ [16763\ \text{km}, 0.5957, 63.4°, 270°, 270°, 0°] \end{cases} \tag{14}$$

where a is the semimajor axis, e is the eccentricity, i is the orbital inclination, Ω is the right ascension of ascending node, ω is the perigee angle, and f is the true anomaly. The perigee height of the relay satellite is about 400 km, and the orbit period is about 0.25 days.

(2) Task Duration Constraint

The initial time of the task is 1 January 2020 00:00:00 (UTC), and the corresponding Julian day is

$$JD_0 = 2458849.5 \tag{15}$$

The three tasks are connected in turn, lasting 240 days in total. The Task 1 starts from the initial time to the end of the last target point, and lasts for 60 days at most; the Task 2 starts on the 61st day and ends at the last target point revisit, lasting for 120 days at most; the Task 3 starts on the 181st day and ends with no fuel, lasting for 60 days at most.

(3) Constraints on Satellite Maneuverability

During the task, the direction of the single impulse of the observation satellite is free, the magnitude of the single impulse is no more than 0.1km/s, and the time interval between two adjacent impulses of each satellite is required to be no less than 0.5 days. The task is considered to be over when the fuel of both observation satellites is exhausted.

(4) Height Constraint

During the whole mission period, the orbital heights of the observation satellites (h) should not be less than 200 km. Otherwise, it is considered that the satellite will crash and the mission fails. In order to ensure the imaging effect of the camera on the target point, the orbital heights of the observation satellites should not exceed 500 km during the observation mission.

(5) Field of View and Solar Altitude Angle

The half field of view of the camera for both observing satellites is $\gamma = 2.5°$. The line of sight of the camera always points to the center of the earth. The observation task will be performed when the target enters the range of the cone angle. Each flight over the target point is only recorded as one observation mission. The observing data by optical satellite are valid only if the solar elevation angle of the target is no lower than 20°. The infrared satellite has no constraint of solar elevation angle.

(6) Data Transmission

Limited by the storage capability on the observing satellites, a data transmission from observing satellite to any relay satellite is needed after observing 30 targets at most. The observation data of the three tasks need to be transmitted within the corresponding task period. The data transmitted beyond the corresponding task period is invalid and no score is calculated. The data transmission is performed when the distance between the observing satellite and the relay satellite is kept within 100 km. The duration of a data transmission is 10 s. If the transmission time is less than 10 s, the transmission fails. Observation missions cannot be performed during transmission.

5 Solution File Format

The file format is summarized in Table 1 and described below (see Attachment 2).

(1) Column 1 is the line number.

(2) Column 2 is the satellite flag: "1" for optical satellite and "2" for infrared satellite.

(3) Column 3 is the action flag: "0" for setting initial orbital parameters, "1" for imple-

menting impulse, "2" for observing targets, and "3" for starting to transmit with relay satellites.

(4) Column 4 is the action time (s), which represents the cumulative seconds from the initial time (1 January 2020 00:00:00), and the accuracy requirement is 10^{-3}.

(5) Columns 5∼7 and Columns 8∼10 are the position components (km) and velocity components (km/s) at the time shown in the same line of Column 4, respectively, and they are expressed in the ECI frame. The accuracy requirements should be within 10^{-3} and 10^{-6}, respectively. If an impulse is performed, Columns 8∼10 provide the velocity components after the impulse.

(6) Columns 11∼13 are the impulse vector (km/s) in the ECI frame and the accuracy requirement is 10^{-6} (can fill 0). If the corresponding action flag (Column 3) is not "1", please fill "0".

(7) Column 14 is the remaining mass (kg) of the satellite, and the accuracy should within 10^{-3}. If an impulse is performed, please fill the mass after the impulse.

(8) Column 15 is the number of the target points. 1∼200 denote the stationary targets, 201∼205 denote the moving targets, and 1001∼1004 denote the relay satellite number (see Attachment 1). Please fill "0" if the action flag is "1", fill the target number if the action flag is "2", and fill the relay satellite number if the action flag is "3".

Notes:

(1) The data of optical satellite and infrared satellite are sorted separately. After all data of the optical satellite are listed, then all data of the infrared satellite are listed.

(2) The data of the two satellites are sorted in increasing order of time.

(3) The initial orbital elements are transformed into position and velocity vectors and then filled in the first line, columns 5-10 for the optical satellite data and the infrared satellite data, respectively.

(4) If multiple actions are executed at the same time, they should be arranged in sequence according to the action flag 0∼3.

(5) In order to avoid excessive cumulative error, it is recommended that the time interval between adjacent rows of submitted results should not exceed 5 days.

Table 1: Solution file format.

Line num	Satellite flag	Action flag	Time(s)	r_x(km)	r_y(km)	r_z(km)	v_x(km/s)	v_y(km/s)	v_z(km/s)	Δv_x(km/s)	Δv_y(km/s)	Δv_z(km/s)	m(kg)	Target point num
1	1	0	0	float	float	float	float	float	float	0	0	0	550.000	0
2	1	1	float	float	float	float	float	float	float	float	float	float	float	0
3	1	2	float	float	float	float	float	float	float	0	0	0	float	1∼205
4	1	1	float	float	float	float	float	float	float	float	float	float	float	0
5	1	2	float	float	float	float	float	float	float	0	0	0	float	1∼205
6	1	1	float	float	float	float	float	float	float	float	float	float	float	0
7	1	2	float	float	float	float	float	float	float	0	0	0	float	1∼205
8	1	3	float	float	float	float	float	float	float	0	0	0	float	1001∼1004
9	2	0	0	float	float	float	float	float	float	0	0	0	550.000	0
10	2	1	float	float	float	float	float	float	float	float	float	float	float	0
11	2	2	float	float	float	float	float	float	float	0	0	0	float	1∼205
12	2	1	float	float	float	float	float	float	float	float	float	float	float	0
13	2	2	float	float	float	float	float	float	float	0	0	0	float	1∼205
14	2	1	float	float	float	float	float	float	float	float	float	float	float	0
15	2	2	float	float	float	float	float	float	float	0	0	0	float	1∼205
16	2	3	float	float	float	float	float	float	float	0	0	0	float	1001∼1004

6 Parameter Values

The parameter values to be used in the competition are shown in Table 2.

Table 2: Parameter values in the competition.

Parameter	Value	Unit
Sea-level gravity acceleration of earth g_0	9.80665	m/s^2
1 day	86400	s
Earth average radius R_E	6378.137	km
Earth rotation rate ω_E	$7.2921158553 \times 10^{-5}$	rad/s
Constant of earth gravitation μ	398600.4415	km^3/s^2
J_2 perturbation coefficient	1.0826269×10^{-3}	
Atmospheric drag coefficient C_d	2.2	
Upwind area of observation satellite s	8	m^2
Specific impulse I_{sp}	300	s
Half cone angle of camera γ	2.5	deg

Appendix A: Satellite Orbital Dynamic Model

A1 J_2 Perturbation

In the ECI frame, the acceleration of J_2 perturbance is

$$\boldsymbol{a}_{J_2} = \frac{3}{2}\frac{\mu J_2 R_E^2}{r^5}\begin{bmatrix} 5\dfrac{xz^2}{r^2} - x \\ 5\dfrac{yz^2}{r^2} - y \\ 5\dfrac{z^3}{r^2} - 3z \end{bmatrix} \tag{16}$$

A2 Atmospheric Drag

In the ECI frame, the acceleration of atmospheric drag is

$$\boldsymbol{a}_d = -\frac{1}{2}C_d\rho\left(\frac{s}{m}\right)v_R\boldsymbol{v}_R \tag{17}$$

where C_d is the atmospheric drag coefficient, s is the upwind area of spacecraft (which is assumed to be a fixed value, see Table 2), m is the mass of spacecraft, and ρ is the atmospheric density. The 1976 standard atmosphere model is used here, and it is only related to the orbital height of the spacecraft (see the Attachment 3). The atmospheric density in the altitude range of 0∼1000 km is given at 0.5 km intervals, with unit as kg/km^3. The orbital height is determined by $h = \|\boldsymbol{r}\| - R_E$. When the orbital height is not an integer multiple of 0.5 km, the atmospheric density at that height can be obtained by linear interpolation method. For instance, the density value at a height of 300.8 km can be obtained by linear interpolation based on the atmospheric densities of 300.5 km and 301 km. $\boldsymbol{v}_R$ is the velocity vector of the center of mass of the spacecraft with respect to the local atmosphere and $v_R = \|\boldsymbol{v}_R\|$,

$$\boldsymbol{v}_R = \boldsymbol{v} - \boldsymbol{\omega}_E \times \boldsymbol{r} = \begin{bmatrix} v_x + \omega_E y \\ v_y - \omega_E x \\ v_z \end{bmatrix} \tag{18}$$

where $\boldsymbol{r} = [x, y, x]^{\mathrm{T}}$ and $\boldsymbol{v} = [v_x, v_y, v_z]^{\mathrm{T}}$ in the ECI frame are the position and velocity vectors, respectively; $\boldsymbol{\omega}_E = [0, 0, \omega_E]^{\mathrm{T}}$ is Earth rotation rate.

A3 Orbital Elements to Position and Velocity Vectors

Given the orbital elements, the eccentric anomaly E is obtained by

$$\tan\frac{f}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2} \tag{19}$$

Then, we can derive $\boldsymbol{P}$ and $\boldsymbol{Q}$ as

$$\boldsymbol{P} = \begin{bmatrix} \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i \\ \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i \\ \sin\omega\sin i \end{bmatrix} \tag{20}$$

$$\boldsymbol{Q} = \begin{bmatrix} -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i \\ -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i \\ \cos\omega\sin i \end{bmatrix} \tag{21}$$

Finally, the position and velocity vectors in the ECI frame are

$$\boldsymbol{r} = a(\cos E - e)\boldsymbol{P} + a\sqrt{1-e^2}\sin E\boldsymbol{Q} \tag{22}$$

$$\boldsymbol{v} = \frac{\sqrt{\mu a}}{r}(-\sin E\boldsymbol{P} + \sqrt{1-e^2}\cos E\boldsymbol{Q}) \tag{23}$$

Appendix B: Relationship Between Latitude and Longitude and Position Vector [1]

B1 Coordinate Transformation Matrix

Ignoring the effects of precession and nutation, the coordinate transformation from the ECI frame to the Earth-Centered Earth-Fixed (ECEF) frame is obtained by

$$\boldsymbol{r}_{\mathrm{ECEF}} = R_Z(\alpha_G)\boldsymbol{r}_{\mathrm{ECI}} \tag{24}$$

The coordinate transformation from the ECEF frame to the ECI frame is

$$\boldsymbol{r}_{\mathrm{ECI}} = R_Z^{\mathrm{T}}(\alpha_G)\boldsymbol{r}_{\mathrm{ECEF}} \tag{25}$$

where,

$$R_Z(\alpha_G) = \begin{bmatrix} \cos\alpha_G & \sin\alpha_G & 0 \\ -\sin\alpha_G & \cos\alpha_G & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{26}$$

and where α_G represents the Greenwich mean sidereal time at the current time,

$$\alpha_G = \alpha_{G0} + \omega_E \Delta t \tag{27}$$

where $\alpha_{G0} = 1.747455428309031$ rad is the initial Greenwich mean sidereal time, and Δt(s) is the duration from the initial time to the current time.

B2 Solve Latitude and Longitude from Position Vector

Given the position vector of $\boldsymbol{r}_{\mathrm{ECEF}} = [x_{\mathrm{ECEF}}, y_{\mathrm{ECEF}}, z_{\mathrm{ECEF}}]^{\mathrm{T}}$ in the ECEF frame, the longitude λ is*

$$\lambda = \arctan\left(\frac{y_{\mathrm{ECEF}}}{x_{\mathrm{ECEF}}}\right) \tag{28}$$

and the latitude ϕ is

$$\phi = \arcsin\left(\frac{z_{\mathrm{ECEF}}}{r_{\mathrm{ECEF}}}\right) \tag{29}$$

where $r_{\mathrm{ECEF}} = \|\boldsymbol{r}_{\mathrm{ECEF}}\|$.

*The longitude and latitude used in the competition is the geocentric longitude and latitude by default.

B3 Solve Position Vector from Latitude and Longitude

Given the latitude and longitude (λ, ϕ) of a ground target point, the position vector $\boldsymbol{r}_{\text{ECEF}} = [x_{\text{ECEF}}, y_{\text{ECEF}}, z_{\text{ECEF}}]^{\text{T}}$ of this point in the ECEF frame is

$$\begin{cases} x_{\text{ECEF}} = R_E \cos\phi \cos\lambda \\ y_{\text{ECEF}} = R_E \cos\phi \sin\lambda \\ z_{\text{ECEF}} = R_E \sin\phi \end{cases} \tag{30}$$

The position vector $\boldsymbol{r}_{\text{ECI}}$ of the target point in the ECI fame can be obtained by Eq. (25).

Appendix C: Solar Altitude Elevation Model [1]

The solar elevation angle is the angle between the direction of the Sun and the local horizontal plane where the ground target is located. Given the latitude and longitude (λ, ϕ) of a certain ground point, the corresponding solar elevation angle $h_\odot$ of this point at a certain time is

$$h_\odot = \arcsin\left[\sin\phi \sin\delta_\odot + \cos\phi\cos\delta_\odot\cos(\alpha_\odot - \alpha)\right] \tag{31}$$

where $\alpha_\odot$ and $\delta_\odot$ are the right ascension and declination of the Sun at the current time, respectively; α is the right ascension of the ground target point. Given the duration Δt(s) from the initial time to the current time, the right ascension of the ground target point is

$$\alpha = \lambda + \alpha_G \tag{32}$$

where, α_G is the Greenwich mean sidereal time by Eq. (27).

The Julian day at the current time is

$$JD = JD_0 + \frac{\Delta t}{86400} \tag{33}$$

where JD_0 is the initial Julian day by Eq. (15). The Julian century of the current time is

$$T_{JD} = \frac{JD - 2451545}{36525} \tag{34}$$

The mean longitude of the Sun and other parameters are defined as follows:

$$\lambda_{M_\odot} = 280.460^\circ + 36000.771 T_{JD} \tag{35}$$

$$M_\odot = \frac{\pi}{180^\circ}(357.5277233^\circ + 35999.05034 T_{JD}) \tag{36}$$

$$\lambda_{ecliptic} = \frac{\pi}{180^\circ}\left[\lambda_{M_\odot} + 1.914666471^\circ \sin M_\odot + 0.019994643\sin(2M_\odot)\right] \tag{37}$$

The obliquity of the ecliptic is defined as

$$\epsilon = \frac{\pi}{180^\circ}(23.439291^\circ - 0.0130042 T_{JD}) \tag{38}$$

Finally, the declination of the Sun at the current time is

$$\delta_\odot = \arcsin(\sin\epsilon \sin\lambda_{ecliptic}) \tag{39}$$

The right ascension of the Sun at the current time can be calculated by

$$\begin{cases} \sin\alpha_\odot = \dfrac{\cos\epsilon\sin\lambda_{ecliptic}}{\cos\delta_\odot} \\ \cos\alpha_\odot = \dfrac{\cos\lambda_{ecliptic}}{\cos\delta_\odot} \end{cases} \tag{40}$$

Reference

[1] Vallado, D. A., *Fundamentals of Astrodynamics and Applications*, 3nd ed., Microcosm Press, Torrance, CA, 2007.